

# BEC - BCS Crossover Part II

## Projection Experiments:

⇒ Projection with just two atoms

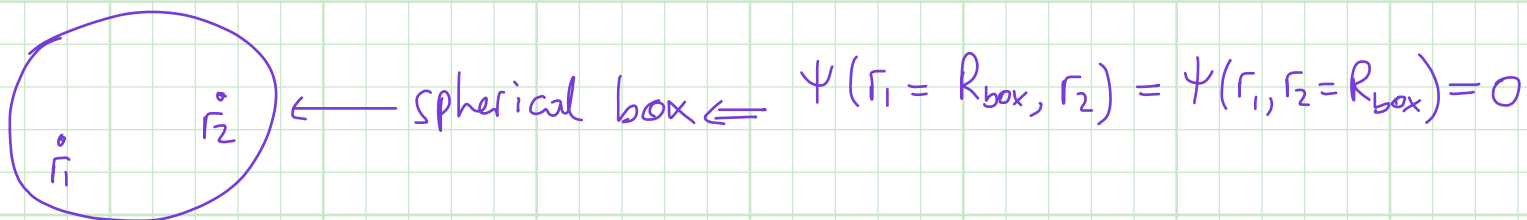
⇒ Generalization to many atoms

Consider two atoms interacting with an attractive potential

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V(r_1 - r_2)$$

Wave function is a function of both  $r_1$  and  $r_2$ :  $\Psi(r_1, r_2)$ .

I want to capture the full spectrum of the atom pair in a spherical box

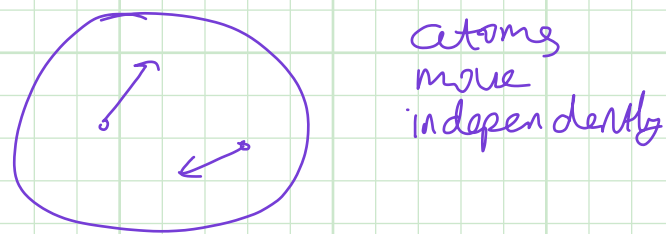
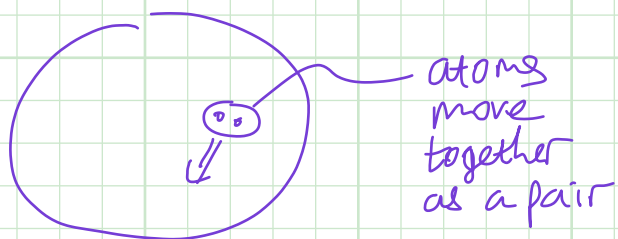


Why place atoms in a box? This way I can get both the bound states and the scattering states, and get the DOS.

⇒ using the states I obtain I can test projection across resonance.

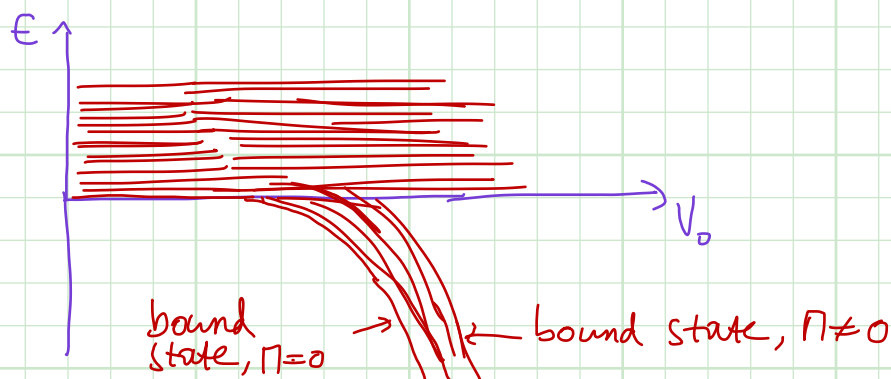
bound States

Scattering States



What do you expect the spectrum to look like as a function of the depth of  $V(r_1 - r_2)$ ? Assume  $V(r_1 - r_2)$  is attractive and short ranged, e.g.  $V(r_1 - r_2) = -V_0 \Theta(-|r_1 - r_2| + R_{\text{box}}/100)$

$$V(r_1 - r_2) = -V_0 \Theta(-|r_1 - r_2| + \underbrace{R_{\text{box}}/100}_{r_e})$$



Solving the 2-particle schrodinger eq in spherical box is hard, even numerically!

Why?

Suppose we discretize space into  $100 \times 100 \times 100$  grid. Since  $\Psi$  is a function of both  $r_1$  and  $r_2$  we need to define  $\Psi(r_1, r_2)$  on

$$(100)^3 \times (100)^3 = 10^{2 \times 3 \times 2} = 10^{12} \text{ points!}$$

The discrete version of the Hamiltonian is a  $10^{12} \times 10^{12}$  matrix! (Although most of the entries in  $H$  are zero) This is a very hard problem indeed.

Simplified version (toy problem)

One way to simplify the problem is to make the boundary "easier."

(1) Let's use COM coordinates

$$R = \frac{r_1 + r_2}{2} \quad \Gamma = r_1 - r_2$$

In these coordinates

$$H = \frac{\Pi^2}{2M} + \frac{p^2}{2\mu} + V(r)$$

$\uparrow \quad \uparrow$   
 $M = 2m \quad \mu = m/2$

(2) Let's focus on  $\Pi = 0$  part  $\Rightarrow$  The point of this approximation is that if initial state has  $\Pi = 0$ , the final state does as well.

(3) Let's set  $\Psi(r = R_{\text{box}}) = 0$ .

$\Rightarrow$  I still want a finite sized box

$\Rightarrow$  Not a very physical B.C.

$\Rightarrow$  very nice for numerics

$\Rightarrow$  obtain qualitatively correct results.

With  $\Pi=0$  and strange boundary condition, we get :

$$\begin{cases} H = \frac{p^2}{2\mu} - V(r) & \Rightarrow \left[ -\frac{\hbar^2}{2\mu} \partial_r^2 - V(r) \right] u(r) = E u(r) \\ \Psi(r=R_{\text{box}}) = 0 & \Rightarrow u(r=R_{\text{box}}) = 0 \end{cases}$$

Solve numerically :

(1) discretize space into steps of  $\delta$

$$\partial_r^2 \Psi(r) \rightarrow \frac{\Psi(r+\delta) - 2\Psi(r) + \Psi(r-\delta))}{\delta^2}$$

$$KE = \frac{-\hbar^2}{2\mu\delta^2} \begin{pmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & -2 & 1 \end{pmatrix} \quad \begin{matrix} \uparrow \\ R_{\text{box}}/\delta \end{matrix}$$

$$PE = \begin{pmatrix} -V_0 & & & & \\ & -V_0 & & & \\ & & \ddots & & \\ & & & -V_0 & \\ & & & & 0 & \dots \end{pmatrix}$$

(2) Solve numerically the eigenvalue problem

$$(KE + PE)\Psi = E\Psi$$

[ I use mathematica's

Eigenvalues[KE+PE] to find the eigenvalues  
 Eigenvectors[KE+PE] to find the eigenvectors ]

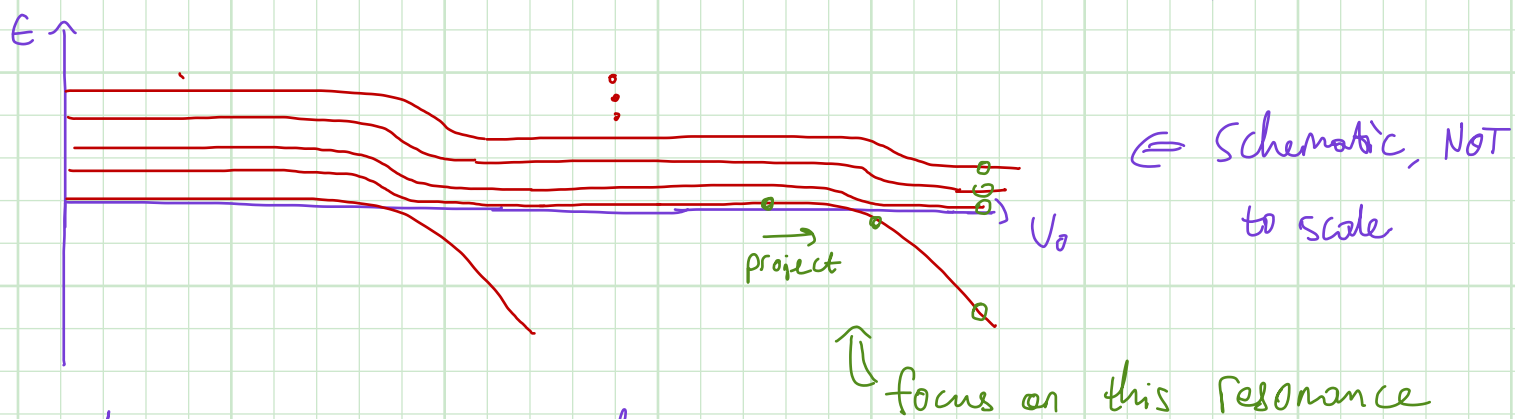
Parameters if you want to try are

$$\frac{\hbar^2}{2\mu\delta^2} = t = 1$$

$$V_0/\delta = 5$$

$$R_{\text{box}}/\delta = 500$$

Plot spectrum as a function of  $V_0$ : ask for expectations



$\Rightarrow$  solve the spectrum using mathematica

⇒ How to find scattering length from our solutions?

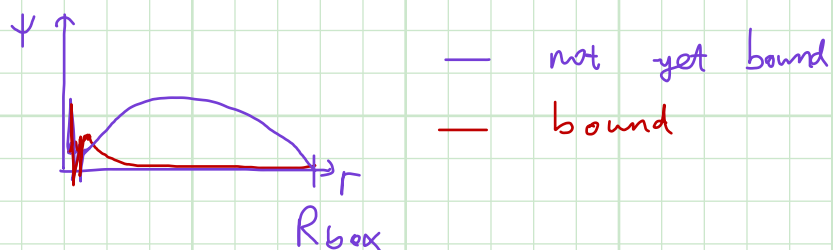
$$\psi(r) = a \sin(k(x-a))$$

$\lim_{x \rightarrow \infty}$

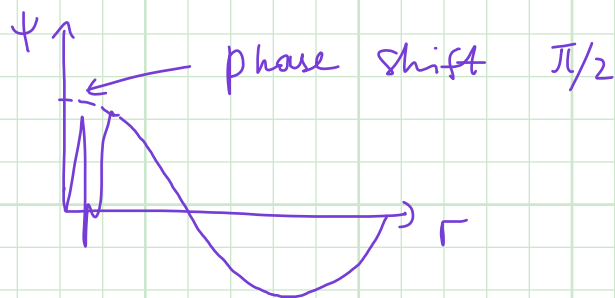
this is the scattering length

⇒ what do we expect the wave functions to do across resonance?

⇒ lowest energy wave function

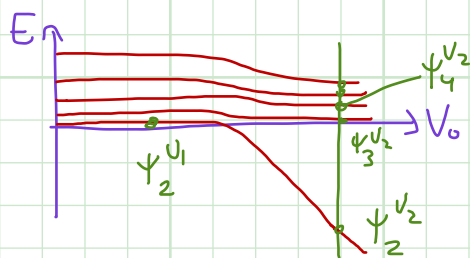


⇒ other wave functions



⇒ Projection experiments

(1) Start to the left of resonance and jump to the right



⇒ find eigenstates at the 2 values of  $V_0$

⇒ compute overlap

$$|\langle \psi_2^u | \psi_k^v \rangle|^2$$

⇒ mostly atoms end up in scattering states

⇒ very low probability to end up in molecular state

(2) Start closer to resonance ⇒ what changes?

(3) Start on-resonance

(4) Start beyond resonance

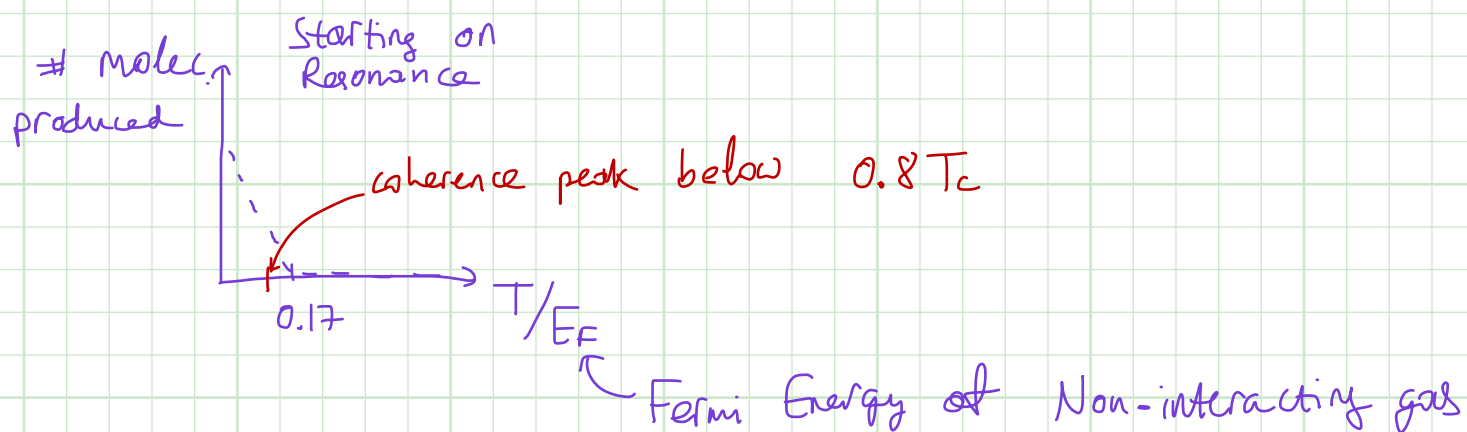
⇒ mostly atoms end up in molecular state

⇒ very low probability to end up in scattering state.

⇒ With 2 atoms weak attraction does not result in pairing

- ⇒ with many atoms weak attraction ⇒ BCS pairing
- ⇒ enhanced probability to make molecules
- ⇒ this tendency is measured by projection experiments

By measuring # and momentum distribution of resulting molecules the manuscript aims to estimate the # and phase coherence of the original pairs.



- ⇒ what is the momentum of the molecules produced?
- ⇒ same as momentum of initial pair
- ⇒ coherence in BCS ⇒ all pairs have total momentum  $+k - k = 0$
- ⇒ coherent pairs will be converted to molecules with zero momentum
- ⇒ "spurious" molecules ⇒ with 2 atom case we saw that there is a finite but small overlap to make molecules even when we start to the left of resonance.
- ⇒ Same tendency occurs in many atom systems.
- ⇒ Total momentum does not have to be zero as  $k$  for each atom can range from 0 to  $k_F$ .
- ⇒ "background" signal at all molecular momenta